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Enhancing Financial Risk Management through LSTM and Extreme Value Theory: A High-Frequency Trading Volume Approach

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Abstract: This study employs high-frequency trading volume data in the financial sector to apply the Long Short-Term Memory (LSTM) model for purposes of risk management. By incorporating high-frequency data, which includes trading volume information, with the LSTM model, we develop an LSTM-RV dynamic prediction model for realized volatility. Employing a semi-parametric Extreme Value Theory (EVT) approach, we estimate return quantiles to construct the efficiency risk management model. Empirical analysis reveals that the LSTM-RV prediction model markedly improves prediction accuracy compared to the traditional Heterogeneous Autoregressive (HAR) volatility prediction model. Additionally, the LSTM-RV-EVT model outperforms both the conventional model and models that exclude trading volume information.

Keywords:LSTM model; EVT method; Value-at-Risk (VaR) risk management; Recurrent Neural Networks

1.Introduction

In recent years, deep learning has become a prominent branch of machine learning, achieving notable success across various domains. This has led to a surge of interest in deep learning theories, methods, and applications both domestically and globally. Common deep learning models include Convolutional Neural Networks (CNN), Recurrent Neural Networks (RNN), Long Short Term Memory (LSTM) networks, Deep Belief Networks (DBN), and Restricted Boltzmann Machines (RBM), among others [1].

Financial risk arises from the volatility of future returns on financial assets, and Value at Risk (VaR) is a widely utilized metric for measuring financial risk. VaR quantifies financial asset returns, typically involving the quantiles of standardized returns and future volatility (standard deviation) of returns. There are three primary approaches to characterizing the distribution of returns: parametric methods, non-parametric methods (such as Monte Carlo simulation and historical simulation, which do not assume any specific return distribution and derive quantiles directly from historical data or simulations), and semi-parametric methods (such as Extreme Value Theory (EVT), which can model the tail risk of VaR). Research by Karmakar et al. [2] has shown that VaR prediction models based on EVT perform better than other models.

Volatility forecasting is a critical aspect of VaR risk management and a popular research topic in finance. Andersen and Bollerslev [3], using high-frequency financial data, introduced Realized Volatility (RV), which effectively measures volatility in high-frequency data. Traditional RV forecasting primarily relies on time series analysis models. Sattarhoff et al. [4] found that the autocorrelation function of daily RV series decays slowly, indicating long memory, and proposed the use of the ARFIMA model to characterize and predict the dynamic changes in RV. Corsi [5] developed the Heterogeneous Autoregressive (HAR) model to capture the long memory of RV and predict future RV. Building on the HAR model, Bollerslev et al. [6] proposed the HARQ and HARF models. Empirical studies have demonstrated that RV prediction models based on high-frequency data have a predictive advantage over traditional low-frequency time series models like GARCH.

The widespread application of machine learning and deep learning in technology has also gained significant attention in the financial field. Barunik and Krehlik [7] suggested using artificial neural networks to predict the volatility of energy markets, highlighting that incorporating high-frequency data characteristics improves volatility prediction accuracy. Chen Weihua and Xu Guoxiang [8] utilized LSTM and stock forum data to predict RV in the high-frequency data domain and compared it with traditional time series models like GARCH, ARFIMA, and HAR, finding that the LSTM model outperformed traditional models in prediction accuracy.

There is limited discussion in the literature on how to construct VaR measurement models using deep learning theory and LSTM models with transaction volume data and apply them to financial risk management. This paper aims to utilize high-frequency price information and transaction volume data to construct the realized volatility of prices and transaction volumes. By integrating LSTM models, we develop an LSTM-RV model for volatility prediction. Combining this with the semi-parametric EVT method for return quantile measurement, we construct the unique financial risk management model and conduct empirical analysis.

2. Related work

2.1. Long Short-Term Memory (LSTM) model

Standard Recurrent Neural Networks (RNNs) are a class of sequence models with short-term memory capabilities. However, during the training process, they often encounter issues with long-term dependencies, resulting in the vanishing gradient problem. In practical applications, the most effective sequence models are gated RNNs, which include Long Short-Term Memory (LSTM) networks and Gated Recurrent Unit (GRU) networks. LSTM networks are more widely used and generally perform better, so this paper focuses on the application of LSTM networks in VaR risk management. Similar to standard RNNs, each hidden layer node in an LSTM has input and output. However, each hidden layer node also includes a system of gated units that control the flow of information. These gated units selectively incorporate new information and forget previously accumulated information. The core component is the memory cell state unit, and the combination of memory cells and gated units effectively addresses the vanishing gradient problem of traditional RNNs. Memory cells, by introducing self-connecting recurrent units, can store long-term information. These units record the state of the memory cell over time and can retain information from the distant past. Let σ denote the sigmoid function, which ranges from 0 to 1 and controls the opening and closing of gated units, and tanh denote the hyperbolic tangent function. LSTM cells can remember past long-term information, and the self-loop weight is controlled by the forget gate, which filters historical information. The forget gate is defined as:

$$g_t = \sigma\left(U_g^T x_t + W_g^T h_{t-1} + b_g \right) \quad (1)$$

Among them, $b\ g$, $U\ g$, and $W\ g$ represent the bias vector, input weight matrix, and recurrent weight matrix of the forget gate, respectively. x_t denotes the feature vector at time step t, h_{t-1} represents the hidden state vector at time step $t-1$, and g t is the output vector of the forget gate at time step t. The cell state is also influenced by the input, and the external input gate vector i_t is defined as:

$$i_t = \sigma\left(U_i^T x_t + W_i^T h_{t-1} + b_i \right) \quad (2)$$

The parameters are defined similarly to those of the forget gate. The input modulation gate vector c_t and the cell state vector c_t are defined as follows:

$$\widetilde{c}_t = \tanh\left(U_c^T x_t + W_c^T h_{t-1} + b_c \right), c_t = g_t \cdot c_{t-1} + i_t \cdot \widetilde{c}_t \quad (3)$$

From equation (3), it is clear that the cell state c_t is governed by two gate units: the forget gate g_t and the input gate i_t. The forget gate g_t regulates the previous cell state c_{t-1}: when the forget gate value is near 0, the previous cell state is almost "forgotten"; when the value is near 1, the previous cell state is fully "retained.". Thus, these two gate units are essential in the cell state update process. The output of the LSTM cell is managed by the output gate. The output gate vector o_t and the final hidden state output vector h_t are defined as follows:

$$o_t = \sigma\left(U_o^T x_t + W_o^T h_{t-1} + b_o \right), h_t = \tanh(c_t) \cdot o_t \quad (4)$$

The parameters are defined similarly to those of the forget gate. Essentially, an LSTM operates through three gate units—namely, the input gate, the forget gate, and the output gate—which control the input to the memory cell, update the cell state, and manage the output of the LSTM cell, respectively. This structure enables the LSTM to "remember" past information over extended periods. The memory cell can retain a portion of information for future use, effectively mitigating the vanishing gradient problem in RNNs.

3.Model Construction

Value at Risk (VaR) refers to the maximum potential loss in the value of a financial asset or portfolio over a specified time period at a given confidence level. Typically, when measuring VaR, it is necessary to standardize daily returns:

$$\eta_t = \frac{R_t}{\sqrt{\sigma_t}}, \eta_t \sim F \quad (5)$$

Where η_t represents the standardized return on day t, R_t represents the negative logarithmic return on day t, and σ_t represents the volatility of returns on day t. F is the distribution function of standardized returns. The $100_{(1-p0)}$% confidence VaR for day t, based on information from day $t-1$, is predicted as follows:

$$VaR_t = F^{-1}(1 - p_0)\sqrt{\sigma_{t|t-1}} \quad (6)$$

Here, $F^{-1}(1-p_0)$ denotes the (1 - p_0) quantile of the distribution function F, and $\sigma_{t|t-1}$ represents the volatility forecast for day t based on information from day $t-1$. Typically, p_0 is set to 0.01 or 0.05; in this paper, p_0 is set to 0.01. Equation (6) provides the VaR for holding a certain asset. If considering short-selling the asset, the standardized return η_t is simply negated, and the VaR for the short-selling scenario is calculated in a similar manner. In this paper, σ_t is measured using realized volatility, and the quantile $F^{-1}(1-p_0)$ is estimated using the threshold model of extreme value theory. A detailed description of the VaR measurement model follows.

3.1.Realized Volatility

Andersen and Bollerslev [3] constructed realized volatility, which is defined as the sum of squared intraday returns on day t. The realized volatility of the price process on day t is defined as follows:

$$RV(p)_t = \sum_{i=1}^{N(\Delta)} \left(\ln p_{t,i+1} - \ln p_{t,i} \right)^2 \quad (7)$$

In addition to using high-frequency trading price data to measure VaR, this paper also utilizes high-frequency trading volume data. Let $V_{t,i}$ represent the trading volume at the $i\ \Delta$

moment on day t. The realized volatility of the trading volume process on day t is defined as follows:

$$RV(V)_t = \sum_{i=1}^{N(\Delta)} \left(\ln V_{t,i+1} - \ln V_{t,i}\right)^2 \tag{8}$$

3.2.Construction of the LSTM-RV Dynamic Volatility Prediction Model

The log-transformed RV approximately follows a normal distribution, which has better statistical properties and aids in improving prediction accuracy. The log-transformed RV exhibits significant long-term memory characteristics. Given that the deep learning LSTM model is well-suited for predicting long-memory time series, this paper utilizes trading price information and trading volume information, using ln $RV(p)$ and ln $RV(V)$ as input variables. The LSTM model is employed to predict $RV(p)$, constructing the LSTM-RV dynamic volatility prediction model as follows:

$$\ln RV(p)_{t+1|t} = LSTM(\ln RV(p)_t, \cdots, \ln RV(p)_{t-t_1}, \ln RV(V)_t, \cdots, \ln RV(V)_{t-t_2}) \tag{9}$$

The model comprises two parts: the logarithmic realized volatility sequence of prices ln $RV(p)$from the past $t_1$days and the logarithmic realized volatility sequence of trading volume ln $RV(V)$ from the past t_2 days. The input variables are processed through a single-layer LSTM model, followed by a fully connected layer, ultimately yielding the predicted value of $RV(p)$ for day t+1. This model offers several advantages: First, modeling the log-transformed RV enhances statistical properties and prediction accuracy. Second, log-transforming volatility ensures that the predicted results are non-negative, which is more realistic. Finally, the proposed model can leverage multi-source information such as trading price and trading volume to predict the realized volatility of prices.

3.3.Construction of the LSTM-RV-EVT Model

From equation (6), it is evident that VaR measurement involves the quantile of standardized returns and volatility. Typically, returns exhibit characteristics such as high peaks and fat tails, necessitating the modeling of their distribution when calculating return quantiles. This paper employs a semi-parametric model, utilizing extreme value theory (EVT) to estimate return quantiles. Parametric models carry the risk of incorrect distributional assumptions, while non-parametric methods suffer from low statistical efficiency. Semi-parametric methods effectively address these shortcomings, achieving a better balance. In practice, the peak-over-threshold method is commonly used in EVT to measure the tail characteristics of returns.

$$F_{\xi,\beta}(x) = \begin{cases} 1-\left(1+\frac{\xi x}{\beta}\right)^{-\frac{1}{\xi}}, \xi \neq 0 \\ 1-\exp\left(1-\frac{x}{\beta}\right), \xi = 0 \end{cases} \tag{10}$$

Where ξ is the shape parameter and β is the scale parameter. When $\xi \geq 0$, the generalized Pareto distribution exhibits heavy tails; when $\xi < 0$, the distribution is truncated. Thus, given a value of p, the quantile estimate can be expressed as:

$$\widehat{F^{-1}}(1-p_0) = u + \frac{\hat{\beta}}{\hat{\xi}}\left[\left(\frac{p_0}{\frac{n_u}{n}}\right)^{-\hat{\xi}} - 1\right] \tag{11}$$

Therefore, by combining equation (3) for the LSTM-RV model and equation (11) for the EVT model, and using the information up to day t, the expression for the VaR value on day t+1 can be derived. The LSTM-RV-EVT model for VaR is constructed as follows:

$$\begin{aligned} VaR_{t+1|t} &= \widehat{F^{-1}}(1-p_0)\sqrt{RV(p)_{t+1|t}} \\ &= \left\{u + \frac{\hat{\beta}}{\hat{\xi}}\left[\left(\frac{p_0}{\frac{n_u}{n}}\right)^{-\hat{\xi}} - 1\right]\right\}\sqrt{RV(p)_{t+1|t}} \end{aligned} \tag{12}$$

The LSTM-RV-EVT Value at Risk (VaR) measurement model has the following advantages: First, it utilizes realized volatility (RV) derived from high-frequency data instead of volatility from classic time series, making implicit volatility explicit and facilitating the creation of a dynamic volatility model. Second, it employs the deep learning LSTM model, integrating multiple data sources such as trading price and volume information to fully exploit data and achieve accurate RV predictions. Finally, the model uses a semi-parametric extreme value theory (EVT) approach to estimate the quantiles of return tails, avoiding the risk of incorrect distributional assumptions associated with parametric models and improving statistical estimation efficiency compared to non-parametric methods.

3.4.VaR Evaluation

After completing the VaR measurement and prediction, it is necessary to evaluate the accuracy of the VaR measure. The CC test combines both the UC test and the IND test. Soltyk et al.[9] developed the generalized method of moments conditional statistic $J_{cc}(q)$ to address the CC test, where q is the degree of freedom. When q=1, $J_{cc}(1)$ is equivalent to the usual likelihood ratio test. This method indicated that this test method is superior to the widely used likelihood ratio test. This paper will use the generalized method of moments conditional test to analyze and compare the effectiveness of VaR models.

4. Empirical Analysis

This paper uses 5-minute high-frequency trading data of the FTSE 100 Index, covering the period from January 4, 2006, to October 31, 2023, totaling 4,299 trading days. The trading data include 5-minute opening prices, closing prices, and trading volumes, with 48 5-minute closing price data points for each trading day. The sample dataset is divided into a training set and a testing set. The training set spans from January 4, 2006, to December 31, 2015, and is used for model parameter estimation. The testing set spans from January 4, 2016, to October 31, 2023, and is used to validate the model constructed in this paper. During model training, 90% of the training set data is used as training samples to estimate the parameters of the LSTM model, while 10% is used as the validation set. The training set interval length is then kept constant while rolling predictions for the next day's price volatility are made using the dynamic LSTM-RV prediction model as described in equation (3). Both t_1 and t_2 are set to 19, meaning that the model uses 20 days of data to predict future realized volatility.

4.1.Statistical Analysis of Data Characteristics

Table 1 on the next page presents the descriptive statistical characteristics of each variable sequence used in this paper. As shown in the table, the skewness of the logarithmic RV and standardized returns is small and exhibits low kurtosis, while the other variable sequences display skewness and high kurtosis. Additionally, the J-B test statistics for each sequence are significant at the 1% significance level, indicating that none of the sequences meet the normal distribution assumption. The Ljung-Box test statistics Q(5)), Q(10), and Q(20) for lags of 5, 10, and 20 periods are all significant at the 1% significance level, suggesting that each sequence exhibits autocorrelation and long-memory characteristics.

Table 1 Descriptive Statistics of Each Variable

Statistic	Mean	StdDev	Skewness	Kurtosis	J-B	Q(5)	Q(10)	Q(20)
RV(p)	0.0003	0.0004	5.687	50.5153	480720.88***	6687***	9785.6***	15463***
RV(V)	4.3469	9.020	9.0847	96.0279	1712533.82***	4710.9***	7209***	10294***
lnRV(p)	9.2709	1.0805	0.3293	-0.115	80.01***	12191***	21841***	38139***
lnRV(V)	1.1050	0.6464	1.7639	6.8362	10612.70***	7973***	14406***	24973***
Standardized Return	-0.112	1.1408	0.0357	-0.2607	12.97***	20.03***	47.15***	101.14***

*** indicates significance at the 1% level; Q(n) is the n-lag Ljung-Box test statistic.

4.2. Comparison of Volatility Prediction Models

First, this paper compares the volatility prediction accuracy of six models using high-frequency data: LSTM-RV, LSTM, HAR, HARQ, HARQF, and ARFIMA. The LSTM-RV model is the proposed model (3), utilizing high-frequency trading price and volume information. The HAR (Heterogeneous Autoregressive) model, proposed by Corsi , sets the regressors as the past 1 day, 5 days, and 22 days of RV, with the dependent variable being the RV for the next day. Bollerslev et al. extended the HAR model by incorporating the realized quarticity (RQ) function into all the regression coefficients, resulting in the HARQF model. If only the past 1-day RV term coefficient is extended to the RQ function, the HARQF model becomes the HARQ model. The ARFIMA model is a fractional integrated moving average model for ln RV, with model order parameters determined by the AIC criterion. This paper compares the performance of these models in predicting RV and evaluating VaR.By comparing the predicted values with the actual values, we obtain the prediction error. This paper uses four common metrics to evaluate prediction accuracy: Mean Squared Error (MSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Gaussian Quasi-Likelihood Error (QLIKE). The expressions for these metrics are as follows:

$$MSE=\frac{1}{N}\sum_{n=1}^{N}\left(y_n-\widehat{y_n}\right)^2,\ MAE=\frac{1}{N}\sum_{n=1}^{N}|y_n-\widehat{y_n}|,$$
$$QLIKE=\frac{1}{N}\sum_{n=1}^{N}\left(\frac{y_n}{\widehat{y_n}}+\ln\frac{y_n^2}{\widehat{y_n}^2}-1\right),\ MAPE=\frac{1}{N}\sum_{n=1}^{N}\frac{|y_n-\widehat{y_n}|}{|y_n|} \tag{13}$$

Table 2 presents the numerical results of six volatility models under four prediction accuracy metrics (the smaller the value, the more accurate the model prediction). As shown in Table 2, the LSTM-RV model has the smallest indicator values across all four evaluation criteria, indicating the best performance. Figure 1 shows the percentage improvement in prediction accuracy of the LSTM-RV model compared to the other five models. Specifically, the LSTM-RV model improved over the HAR model by 16.15%, 23.79%, 22.09%, and 21.94% in MSE, MAE, QLIKE, and MAPE, respectively. From Table 2 and Figure 1, we can conclude the following:

1. The LSTM-RV and LSTM models, based on deep learning, show improved performance across all metrics compared to the HAR and ARFIMA models. This indicates that the LSTM-RV and LSTM models are superior to the four classical time series models in terms of prediction accuracy and long-memory characteristics.
2. The proposed LSTM-RV model outperforms the original LSTM model across all metrics, indicating that incorporating trading volume information into the LSTM model improves its prediction performance.

Table 2 Comparison of Prediction Accuracy of Six Volatility Models

Model	MSE	MAE	QLIKE	MAPE
LSTM-RV	4.246	0.65	0.171	0.403
LSTM	4.417	0.664	0.182	0.416
HAR	4.932	0.816	0.20	0.488
HARQ	5.007	0.802	0.201	0.488
HARQF	5.016	0.794	0.201	0.49
ARFIMA	4.665	0.662	0.182	0.487

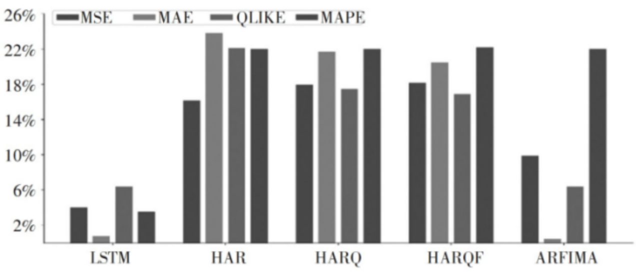

Figure 1. Improvement Percentage in Prediction Accuracy of LSTM-RV Compared to Five Volatility Prediction Models

4.3.Empirical Findings of VaR Measurement Models

Table 3 presents the empirical results for 18 VaR measurement models. The LSTM-RV-EVT model, proposed in this paper, is among these models. The first column of Table 3 lists the names of the VaR measurement models. The second and fifth columns show the actual default rates for holding and short-selling assets, respectively (with a theoretical default rate of 0.01 as assumed in this study). The third and sixth columns show VaR conditional coverage statistic Jcc(q) for holding and short-selling assets, respectively (where q = 5). The fourth and seventh columns rank the effectiveness of the VaR measurement models, with lower ranks indicating superior predictive performance. Rows three to eight provide the empirical results for quantile estimation using the semiparametric EVT method. Rows nine to fourteen present the results using the commonly applied skewed t-distribution (SKST) method. Rows fifteen to twenty summarize the empirical results from the historical simulation method.

Table 3 VaR Test Results for Holding and Short-Selling Asset Scenarios

Model	Held Assets Default Rate	Held Assets Jcc(5)	Held Assets Rank	Short Assets Default Rate	Short Assets Jcc(5)	Short Assets Rank
LSTM-RV-EVT	0.0143	0.4602	1	0.0102	0.9505	1
LSTM-EVT	0.0178	0.0859	10	0.0167	0.1478	12
HAR-EVT	0.0148	0.12	9	0.0116	0.8290	2
HARQ-EVT	0.0183	0.0266	14	0.0145	0.468	9
HARQF-EVT	0.0189	0.0224	15	0.0156	0.266	10
ARFIMA-EVT	0.0210	0.0028	18	0.0188	0.0384	13
LSTM-RV-SKST	0.0124	0.2810	6	0.009	0.6765	8
LSTM-SKST	0.0183	0.0315	12	0.0156	0.1679	11
HAR-SKST	0.0124	0.4488	3	0.0087	0.8498	4
HARQ-SKST	0.0162	0.3447	4	0.0087	0.7692	6
HARQF-SKST	0.0157	0.2694	7	0.008	0.9396	3
ARFIMA-SKST	0.0181	0.0457	11	0.0135	0.3905	7
LSTM-RV-H	0.015	0.3381	4	0.0101	0.9504	1
LSTM-H	0.0182	0.0309	12	0.0166	0.1477	11
HAR-H	0.0161	0.1361	7	0.0117	0.8291	4
HARQ-H	0.0187	0.014	15	0.0144	0.467	7
HARQF-H	0.0193	0.012	16	0.0155	0.265	9

ARFIMA-H	0.0209	0.0027	17	0.0187	0.0383	12

Note: For all statistics except the default rate, the model results are judged by the size of the p-value. A larger p-value indicates a better model. Statistics that fail to pass the test at the 1% significance level are underlined.

From Table 3, the following conclusions can be drawn: (1) Under both long and short asset holding conditions, it achieves the highest p-value, significantly surpassing other models, and passing the test. This demonstrates that the LSTM-RV-EVT model is significantly superior to other VaR measurement models. (2) For short asset holdings, the default rate and VaR empirical results for each model are better than for long asset holdings, indicating an asymmetry in the tail distribution of standardized returns. (3) When combining VaR measurement and volatility prediction results, the ARFIMA model outperforms the HAR model in volatility prediction. However, it underperforms in predicting extreme risks. Conversely, the HAR model, while poorer in overall volatility prediction, excels in predicting extreme risks. Since VaR risk is primarily determined by extreme risks, the HAR model is preferred over the ARFIMA-EVT model in the domain of VaR measurement.

5.Conclusion

This paper examines VaR financial risk management through the lens of deep learning theory. By leveraging high-frequency trading volume data, we developed an LSTM-RV dynamic volatility prediction model. Utilizing the EVT semi-parametric approach to estimate the quantiles of standardized returns, we formulated an LSTM-RV-EVT risk management model and compared its performance to models such as HAR-EVT, HARQ-EVT, HARQF-EVT, ARFIMA-EVT, and LSTM-EVT. The findings are as follows: (1) The LSTM-RV volatility prediction model, which integrates trading volume data and LSTM-based deep learning, provides more accurate predictions than the original LSTM model and long-memory time series models. (2) The p-values for VaR risk measurement models are significantly higher for short asset holdings than for long holdings, indicating asymmetry in return distributions. (3) The proposed risk measurement model, combining the LSTM-RV model with semi-parametric extreme value theory, achieves the most precise VaR risk measurements for both long and short asset holding periods, surpassing the HAR-EVT, HARQ-EVT, HARQF-EVT, ARFIMA-EVT, and LSTM-EVT models. (4) Regarding volatility prediction and VaR measurement, the ARFIMA model outperforms the HAR model in volatility prediction but is less effective in forecasting extreme risk volatilities, making the HAR model more suitable for VaR risk measurement. The prediction model and risk measurement model developed in this study significantly enhance the accuracy of volatility prediction and VaR risk measurement compared to traditional time series models, demonstrating the predictive strengths of deep learning theory.

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